Odd triplet superconductivity in a superconductor/ferromagnet structure with a narrow domain wall

A. F. Volkov[1,2] and K. B. Efetov[1,3]

[1]*Theoretische Physik III, Ruhr-Universität Bochum, D-44780 Bochum, Germany*
[2]*Institute for Radioengineering and Electronics, Russian Academy of Sciences, 11-7 Mokhovaya Street, Moscow 125009, Russia*
[3]*L. D. Landau Institute for Theoretical Physics, RAS, 119334 Moscow, Russia*

We study proximity effect in superconductor/ferromagnet (SF) structure with a narrow domain wall (DW) at the SF interface. The width of the domain wall is assumed to be larger than the Fermi wavelength but smaller than other characteristic lengths (for example, the "magnetic" length). The transmission coefficient is supposed to be small so that we deal with a weak proximity effect. Solving the linearized Eilenberger equation, we find analytical expressions for quasiclassical Green's functions. These functions describe the short-range (SR) condensate components, singlet and triplet, with zero projection of the total spin on the quantization z axis, induced in ferromagnet (F) due to the proximity effect as well as long-range odd triplet component (LRTC) with a nonzero projection of the total spin of Cooper pairs on the z axis. The amplitude of the LRTC essentially depends on the product $h\tau$ and increases with increasing the exchange energy h (τ is the elastic-scattering time). We calculate the Josephson current in superconductor/ferromagnet/superconductor junction with a thickness of the F layer much greater than the penetration length of the SR components. The Josephson critical current caused by the LRTC may be both positive and negative depending on chirality of the magnetic structure in F. The density of states (DOS) in a diffusive SF bilayer is also analyzed. It is shown that the contributions of the SR and LR components to the DOS in F have a different dependence on the thickness d of the F layer (nonmonotonous and monotonous).

I. INTRODUCTION

For a long time the mechanism of superconductivity of Bardeen, Cooper, and Schrieffer (BCS)[1,2] based on the assumption of *s*-wave singlet pairing remained sufficient for explanation of properties of almost all existing superconductors. However, the situation has changed in the last two decades. It has been established that in high T_c superconductors the *d*-wave singlet pairing is responsible for the superconductivity.[3] A triplet *p*-wave pairing was suggested to explain properties of materials with heavy fermions.[4] Recent intensive studies of the strontium ruthenate (Sr_2RuO_4) showed that superconductivity in this compound was also due to the triplet *p*-wave pairing mechanism.[5,6] The *p*-wave pairing was considered to be an essential ingredient for forming the triplet Cooper pairs because, in contrast to the conventional singlet pairing, it allowed to satisfy the Pauli principle.

A more exotic type of triplet pairing was proposed by Berezinskii[7] as a possible mechanism of superfluidity of He³. According to this suggestion the triplet pairing might have a singlet space symmetry. The wave function of the Cooper pairs, $f_{\uparrow\uparrow}(t,t') \propto \langle \psi_\uparrow(t)\psi_\uparrow(t') \rangle$, suggested by Berezinskii[7] was symmetric in both momentum and spin spaces. At the same time, in order to fulfill the Pauli principle, the function $f_{\uparrow\uparrow}(t,t)$ taken at equal times must be zero. The only possibility to satisfy all these requirements is to assume that the wave function in the Matsubara representation $f_{\uparrow\uparrow}(\omega)$ should be an odd function of ω so that $f_{\uparrow\uparrow}(t,t) \propto \Sigma_\omega f_{\uparrow\uparrow}(\omega) = 0$. This is exactly what was suggested by Berezinskii[7] and one may call such a state as odd triplet superconductivity (superfluidity).

Unfortunately, this type of pairing was not more than a hypothesis and no microscopic model leading to the odd trip-

let superconductivity was suggested in Ref. 7. Moreover, it turned out later that in superfluid He³ another type of pairing was responsible for the superfluidity[8,9] and the scenario for the odd triplet superconductivity remained justified neither theoretically nor experimentally.

It was discovered only recently[10] that the odd triplet superconductivity could be realized in a simple system consisting of an ordinary BCS superconductor (S) and ferromagnet (F) with a nonhomogeneous magnetization *M* (for details, see also a review[11] and references therein). In the case of an superconductor/ferromagnet (SF) system (for example, an SF bilayer) with a homogeneous magnetization in F, two types of the condensate arise in the system—a singlet component with a condensate wave function f_3 and a triplet component f_0 with the zero projection of the total spin of the Cooper pair on the quantization axis, $S_z=0$.[11,12] Both the components decay in F over a short length,

$$\xi_h = \sqrt{D/h}, \qquad (1)$$

in the diffusive limit ($h\tau \ll 1$) and over the mean free path $l=v\tau$ in the limit $h\tau \gg 1$, where $D=vl/3$ is the diffusion coefficient and h is the exchange energy. Since the exchange energy h is much larger than the temperature T, the decay length ξ_h is much shorter than the depth of the condensate penetration into a nonmagnetic metal N in the case of an SN system.

In Ref. 10 a diffusive SF system with a domain wall (DW) at the SF interface was considered. It was shown that in this case not only the singlet and triplet $S_z=0$ components but also a triplet $|S_z|=1$ component arises in the system. The triplet component with a nonzero projection of the total spin

S_z penetrates the ferromagnet over a length ξ_ω that does not depend on the exchange energy h and equals

$$\xi_\omega = \sqrt{D/2\omega} \qquad (2)$$

in the diffusive limit, where $\omega = \pi T(2n+1)$ is the Matsubara frequency. This triplet component was called a long-range triplet component (LRTC).

In order to carry out calculation of physical quantities explicitly, it was assumed in Ref. 10 that the width of the domain wall w was much larger than the mean free path l and that the proximity effect was weak due to a nonideal SF interface (the reflection coefficient at the interface R is close to 1). The magnetization vector **M** in the domain wall was assumed to rotate linearly with the coordinate x normal the SF interface so that the angle α between **M** and z axis was equal to $\alpha(x) = Qx$ in the interval $\{0, w\}$ and $\alpha = Qw$ at $x > w$. In this case the condensate functions in F could be found exactly from the linearized Usadel equation.

As a result, the amplitude f_1 of the odd triplet $|S_z| = 1$ component has been obtained in Ref. 10 explicitly. Using the known value of the function f_1, the conductance variation δG of the ferromagnet as a function of temperature T has been determined. It turned out that, in contrast to the superconductor/normal (SN) system where the conductance variation has a maximum at some temperature (a reentrant behavior),[13–17] the function $\delta G(T)$ in the SF system decreased monotonously with increasing temperature.

In subsequent works[18–23,40] the idea about the generation of the odd triplet condensate with the nonzero projection and long-range penetration into the ferromagnets was discussed using somewhat different models and approaches. Kadigrobov *et al.*[18] also considered a model with a DW at the SF interface but, in contrast to Ref. 10, assumed that the length of the DW was shorter than the mean free path. Although nothing was said in the paper about the type of the SF interface, they considered apparently the limit of the ideal SF interface (the transmission coefficient did not enter the equations presented in that paper). It was assumed that in the region of the domain wall the superconducting condensate had to be described by an Eilenberger equation. At distances exceeding the mean free path, one should have the Usadel equation and the Eilenberger equation might be used to derive a boundary condition for the Usadel equation.

Without presenting a solution of the Eilenberger equation, Kadigrobov *et al.*[18] displayed in a simple form an effective boundary condition for the linearized Usadel equation. This boundary condition introduced the triplet condensate as a solution of the Usadel equation and the latter was used to determine the contribution to the conductivity due to the triplet condensate penetrating the ferromagnet over long distances. Another approach to finding the odd triplet component was suggested in Refs. 19, 20, and 24. In that approach the properties of the SF interface are characterized by a scattering matrix the elements of which may be considered as phenomenological parameters. In this approach one does not need knowing the detailed structure of the SF interface and can proceed calculating physical quantities using these parameters. The amplitude of the condensate wave functions has been determined in these papers numerically. Analytical

results were obtained in the framework of this approach in a recent paper,[21] where a ballistic superconductor/ferromagnetic/superconductor (SFS) system was considered. However, from the physical point of view this approach is equivalent to introducing a thin domain wall. If one wants to know details of how the triplet component is generated one has to solve again the microscopic Eilenberger equation for a certain configuration of the magnetic moment, which is similar to considering the model of Ref. 18.

In this paper, we reconsider the problem of the generation of the odd triplet component by a thin domain wall located at the SF interface. We assume that the size the domain wall exceeds the interatomic distances, which allows us to use the quasiclassical Eilenberger equation.[25,26] Below, we solve the Eilenberger equation assuming a weak proximity effect and show that some effective boundary condition for the Usadel equation can indeed be written. However, the results we obtain disagree with those in Ref. 18. It turns out that, in contrast to the formula in Ref. 18, the effective boundary condition for the Usadel equation crucially depends on the relation between the exchange energy h, elastic-scattering rate τ^{-1}, and other parameters. Moreover, the absence of the transmission coefficient $T(\mu)$ in the formulas in Ref. 18 makes us to suppose that the SF interface was assumed to be ideal. However, in this case one has to solve nonlinearized Eilenberger equation and we believe that this can be done only numerically. Therefore we suspect that the form of the boundary condition presented in Ref. 18 is not well justified.

By now, several attempts to observe this new type of the condensate—odd triplet component—have been undertaken. In a recent work[27] the dc Josephson effect has been measured in an SFS Josephson junction consisting of two superconductors (Nb) and the ferromagnet CrO_2 where free electrons have only one direction of spins. The Josephson critical current has been observed in junctions with a separation between S electrodes of about 1 μm. Obviously, the Josephson coupling between the superconductors may only be due to the LRTC. In Ref. 28 a conductance variation δG was measured in an Al/Ho system below the critical temperature of Al. The order of magnitude of the observed change in the conductance can really be explained in terms of the LRTC. In this ferromagnet, a magnetic inhomogeneity is natural because Ho is a helicoidal ferromagnet such that the magnetization vector rotates in space forming a spiral with the period of $\approx 60 A$.

Already earlier experiments on SF structures have also brought an evidence in favor of the existence of a condensate penetrating into the ferromagnet over a long distance.[29–32] It is also worth mentioning that in recent experiments on SFS junctions,[33–36] where the sign reversal of the critical current (π state) has been detected, the magnetization was not homogeneous, and therefore, the triplet component had also to exist and contribute to the critical current. The problem of the triplet component in multilayered SFS junctions[37–41] and in junctions with Neel's domain walls[42] was studied in recent theoretical papers. It was shown, in particular, that the LRTC may also lead to a nonmonotonous dependence of the critical Josephson current on SFS junctions.[41,43] In order to succeed in searching the LRTC in SF structures, it is very important to use materials that might give a large amplitude of the
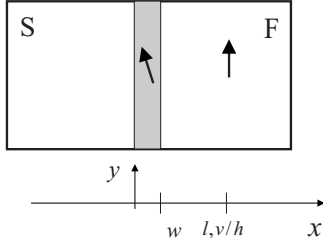
FIG. 1. Schematic picture of a SF bilayer with a domain wall (the shadowed stripe) at the SF interface. The width of the DW is w; $v/h, l$ denote the "magnetic" length and the mean free path.

LRTC. Therefore, it is desirable to have analytical formulas for the amplitude of the LRTC in a wide range of parameters characterizing the system. Calculation of these amplitudes is the ultimate goal of the present work.

The paper is organized as follows. In Sec. II we formulate the model and solve the linearized Eilenberger equation. Expressions for short-range (SR) and long-range (LR) condensate components induced in F are also presented there. In Sec. III spatial dependencies of the SR and LR components are found for weak ($h\tau \ll 1$) and strong ($h\tau \gg 1$) ferromagnets. The Josephson current in a long SFS junction originating from the LRTC is calculated in Sec. IV. In Sec. V we analyze a diffusive SF bilayer with a DW the width of which exceeds the mean free path but is shorter than the "magnetic" length ξ_h. The influence of the spin-dependent scattering on the LRTC is also analyzed in this section. In Sec. VI we consider a diffusive SF bilayer. We calculate the contributions to the density of states (DOS) due to the SR and LR components and discuss a possible reason for an anomalous behavior of the DOS observed in a recent experiment.[44] In Sec. VII we discuss the results obtained.

II. MODEL AND BASIC EQUATIONS

We consider an SF structure with a thin domain wall at the SF interface (see Fig. 1). The thickness of the DW, w, is supposed to be larger than the Fermi wavelength but smaller than all other characteristic lengths (the mean free path $l = v\tau$, the "exchange length" v/h, etc). Outside the interval $\{0, w\}$ the magnetization vector $\mathbf{M}$ in F is parallel to the z axis but inside the domain wall it has a projection on the y axis $M \sin \alpha(x)$. The average $\langle \sin \alpha(x) \rangle_w \equiv \frac{1}{w}\int_0^w dx \sin \alpha(x) \neq 0$ is assumed to differ from zero. The transmission of electrons through the SF interface is supposed to be small so that we deal with a weak proximity effect. This assumption seems to correspond to experiments because even in the absence of a potential barrier at the SF interface the reflection of electrons at the interface is strong due to a considerable mismatch of the Fermi surfaces in the superconductor and ferromagnet.

Our calculations are based on the Eilenberger equation for quasiclassical Green's functions.[25,26,45] In the limit of the weak proximity effect considered here, these functions in S and F deviate weakly from their values in the absence of the contact between S and F. We are interested here in the condensate wave functions induced in F. Due to the presence of

the exchange field acting on spins of free electrons, the system should be described by quasiclassical Green's functions $\check{g}$ that are 4×4 matrices in the particle-hole and spin spaces.

The Eilenberger equation in the ferromagnet F has the form[11,25,26,45]

$$v \nabla \check{g} + \omega[\hat{\tau}_3 \otimes \hat{\sigma}_0, \check{g}] + i[\mathbf{h}(x)\mathbf{S}, \check{g}] + (1/2\tau)[\langle \check{g} \rangle, \check{g}] = 0, \tag{3}$$

where $\mathbf{h}(x) = h[0, \sin \alpha(x), \cos \alpha(x)]$ is the vector of the exchange field, $\omega = \pi T(2n+1)$ is the Matsubara frequency, v is the Fermi velocity, $\mathbf{S} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\tau}_3 \otimes \hat{\sigma}_3)$, $\hat{\sigma}_k$ are the Pauli matrices, and $\hat{\sigma}_0$ is the unit matrix. The square and angular brackets mean the commutator and averaging over angles, respectively, and τ is the elastic-scattering time. The matrices $\hat{\tau}_k$ and $\hat{\sigma}_k$ operate in the particle-hole and spin spaces. As has been assumed previously, outside of the DW ($x > w$) the angle α is just zero, $\alpha(x) = 0$. At the same time, the concrete spatial dependence of the angle α inside the DW is not essential. It is important only that

$$\langle \sin \alpha(x) \rangle_w \equiv \frac{1}{w}\int_0^w dx \sin \alpha(x) \neq 0.$$

Equation (3) is complemented by a boundary condition[46]

$$\check{a} \equiv (\check{g}(\mu) - \check{g}(-\mu))/2 = \text{sgn } \mu \cdot (T(\mu)/4)[\check{g}, \check{g}_S], \tag{4}$$

where $\check{a}$ is the part of the quasiclassical Green's function $\check{g}$ antisymmetric in the momentum space, $\mu = p_x/p$, $\check{g} = \check{g}_F$ is the Green's function in F, and $T(\mu)$ is the coefficient of transmission of electrons through the SF interface which is supposed to be small.

Due to the weakness of the proximity effect, it is convenient to represent the quasiclassical Green's function $\check{g}$ in the form

$$\check{g} = \text{sgn } \omega \cdot \hat{\tau}_3 \otimes \hat{\sigma}_0 + \check{f}, \tag{5}$$

where the first term is the Green's function of the ferromagnet in the absence of the superconducting condensate and the second term is the condensate function we are interested in. Substituting $\check{g}$, Eq. (5), into Eq. (3) and linearizing the Eilenberger equation with respect to $\check{f}$, we come to the equation

$$\text{sgn } \omega \cdot \mu\hat{\tau}_3 \otimes \partial \check{f}/\partial \tilde{x} + (1 + 2|\omega|\tau)\check{f} - i(h_\omega \tau)\cos \alpha(x)[\hat{\sigma}_3, \check{f}]_+$$

$$= \langle \check{f} \rangle + i(h_\omega \tau)\sin \alpha(x)\hat{\tau}_3 \otimes [\hat{\sigma}_2, \check{f}], \tag{6}$$

where $\tilde{x} = x/l$ is the dimensionless coordinate $h_\omega = h \cdot \text{sgn } \omega$, the brackets $[..,..]_+, [..,..]$ stand for the anticommutator and commutator, respectively, and the angular brackets mean the averaging over angles. When writing Eq. (6), we used the fact that the condensate matrix function $\check{f}$ is off-diagonal in the particle-hole space and therefore anticommutes with the matrix $\hat{\tau}_3$. We neglect here the spin-dependent scattering caused by fluctuations of magnetic moments in space and spin-orbit interaction. The influence of this scattering will be discussed in Sec. V.

For small values of the condensate function $\check{f}$, the boundary condition, Eq. (4), can also be linearized and written as

$$\check{a} = \text{sgn } \mu \cdot \text{sgn } \omega \cdot (T(\mu)/2)\hat{\tau}_3 \check{f}_S, \tag{7}$$

where $\check{f}_S = \hat{\sigma}_3 \otimes \hat{\tau}_2 f_S$ is the condensate matrix function in S in the absence of the proximity effect, $f_S = \Delta/\sqrt{\omega^2 + \Delta^2}$.

So, we have to solve Eq. (6) with the boundary condition, Eq. (7). To find the solution we represent the matrix $\check{f}$ as the sum of symmetric $\check{s}$ and antisymmetric $\check{a}$ in the momentum space parts,

$$\check{f} = \check{s} + \check{a}. \tag{8}$$

Substituting the representation for the matrices $\check{s}$ and $\check{a}$, Eq. (8), into Eq. (6), we come to the following equations:

$$\text{sgn } \omega \cdot \mu \hat{\tau}_3 \otimes \partial \check{s}/\partial \tilde{x} + \kappa_\omega \check{a} - i(h_\omega \tau)\cos \alpha(x)[\hat{\sigma}_3, \check{a}]_+$$
$$= i\hat{\tau}_3 (h_\omega \tau)\sin \alpha(x)[\hat{\sigma}_2, \check{a}], \tag{9}$$

$$\text{sgn } \omega \cdot \mu \hat{\tau}_3 \otimes \partial \check{a}/\partial \tilde{x} + \kappa_\omega \check{s} - i(h_\omega \tau)\cos \alpha(x)[\hat{\sigma}_3, \check{s}]_+$$
$$= \langle \check{s} \rangle + i\hat{\tau}_3 (h_\omega \tau)\sin \alpha(x)[\hat{\sigma}_2, \check{s}], \tag{10}$$

where $\kappa_\omega = 1 + 2|\omega|\tau$.

If we neglected the right-hand side, the solution of Eqs. (9) and (10) with the boundary condition (7) would contain only the singlet and $S_z = 0$ triplet components. The presence of the right-hand side of Eqs. (9) and (10) results in the appearance of the LRTC. If the domain width w is small in comparison with the other characteristic lengths of the problem, v/h and l, all the functions vary slowly over this distance. Therefore, we can integrate Eq. (10) over the interval $\{0, w\}$ and obtain an effective boundary condition for the matrix $\check{a}$,

$$\mu \check{a}|_{x=0} = \text{sgn } \omega \cdot b_\mu \hat{\tau}_3 \otimes \hat{\sigma}_3 \hat{f}_S + iH[\hat{\sigma}_2, \check{s}(0)], \tag{11}$$

where $b_\mu = (T(\mu)|\mu|/2)$, $\hat{f}_S = f_S \hat{\tau}_2$, and $H = (h\tau)(w/l)\langle \sin \alpha(x) \rangle_w \equiv (h\tau)(\bar{w}/l)$, with $\bar{w} = w\langle \sin \alpha(x) \rangle_w$. For example, in the case of DW with a linearly varying magnetization, we obtain $\bar{w} = w(2/\pi) \approx 0.64w$.

Now the problem is reduced to solving Eqs. (9) and (10) outside the domain wall ($\alpha = 0$) with the boundary condition (11). We will see that the symmetric part, $\check{s}$, has the following structure in the spin space:

$$\check{s} = (\hat{s}_3 \hat{\sigma}_3 + \hat{s}_0 \hat{\sigma}_0) + \hat{s}_1 \hat{\sigma}, \tag{12}$$

with $\hat{s}_3 = s_3 \hat{\tau}_2$, $\hat{s}_0 = s_0 \hat{\tau}_2$, and $\hat{s}_1 = s_1 \hat{\tau}_1$.

First, we consider elements of the $\check{s}$ matrix diagonal in the spin space. From Eq. (9) we find for $a_\pm \equiv a_{11(22)}$

$$-\hat{a}_\pm \cdot \kappa_{h\pm} = \text{sgn } \omega \cdot |\mu|\hat{\tau}_3 \cdot \partial \hat{s}_\pm/\partial \tilde{x}, \tag{13}$$

where $\kappa_{h\pm} = 1 + 2|\omega|\tau \mp 2ih_\omega \tau$.

Using Eq. (13) the effective boundary condition, Eq. (11), can be written as

$$-\mu^2 \partial \hat{s}_\pm/\partial \tilde{x} = \pm \kappa_{h\pm}[b_\mu \hat{f}_S + 2H_\omega \cdot \hat{\tau}_3 \cdot \hat{s}_1(0)], \tag{14}$$

where $H_\omega = H \cdot \text{sgn } \omega$.

Substituting $a_\pm$ from Eq. (13) into Eq. (10), one can write an equation for $\hat{s}_\pm$ in the form

$$-\mu^2 \partial^2 \hat{s}_\pm/\partial \tilde{x}^2 + \kappa_{h\pm}^2 \hat{s}_\pm = \kappa_{h\pm}\langle \hat{s}_\pm \rangle \pm 2\delta(\tilde{x})\kappa_{h\pm}[b_\mu \hat{f}_S$$
$$+ 2H_\omega \hat{\tau}_3 \cdot \hat{s}_1(0)]. \tag{15}$$

The boundary condition, Eq. (14), is taken into account with the help of the last term on the right-hand side of Eq. (15) and a formal symmetric continuation of the solution to the interval $\{-\infty, 0\}$. Performing the same procedure, we can obtain an equation for $\hat{s}_1$,

$$-\mu^2 \partial^2 \hat{s}_1/\partial \tilde{x}^2 + \kappa_\omega^2 \hat{s}_1 = \kappa_\omega \langle \hat{s}_1 \rangle - 4\delta(\tilde{x})\kappa_\omega H_\omega \hat{\tau}_3 \cdot \hat{s}_3(0) \tag{16}$$

Equation (16) can easily be solved in the same way as it was done for the case of a homogeneous magnetization.[47] For the Fourier transforms $\hat{S}_\pm(k)$ and $\hat{S}_1(k)$ of the functions $\hat{s}_\pm$ and $\hat{s}_1$, we obtain

$$\hat{S}_\pm(k) = \pm 2\frac{\kappa_{h\pm}}{M_{h\pm}(k, \mu)}\left\{ \frac{\kappa_{h\pm}}{1 - \kappa_{h\pm}\langle M_{h\pm}^{-1}(k, \mu)\rangle} \right.$$
$$\left. \times \left\langle \frac{b_\mu \hat{f}_S + H_\omega \hat{\tau}_3 \cdot \hat{s}_1(0)}{M_{h\pm}(k, \mu)} \right\rangle + b_\mu \hat{f}_S + H_\omega \hat{\tau}_3 \cdot \hat{s}_1(0) \right\}, \tag{17}$$

$$\hat{S}_1(k) = -4\frac{H_\omega \kappa_\omega}{M_\omega(k, \mu)}\hat{\tau}_3$$
$$\times \left\{ \frac{\kappa_\omega}{1 - \kappa_\omega \langle M_\omega^{-1}(k, \mu)\rangle}\left\langle \frac{\hat{s}_3(0)}{M_\omega(k, \mu)} \right\rangle + \hat{s}_3(0) \right\}, \tag{18}$$

where $M_{h\pm}(k, \mu) = (k\mu)^2 + \kappa_{h\pm}^2$, $M_\omega(k, \mu) = (k\mu)^2 + \kappa_\omega^2$, $H_\omega = \text{sgn } \omega (h\tau)(w/l)\langle \sin \alpha \rangle_w \equiv \text{sgn } \omega (h\tau)(\bar{w}/l)$, $\kappa_{h\pm} = 1 + 2|\omega|\tau \mp ih_\omega \tau$, and $\kappa_\omega = 1 + 2|\omega|\tau$.

One can see from Eq. (18) that the characteristic length of the decay of the LRTC, $s_1(x)$, does not depend on the exchange energy h.[10] We will see that the spin-dependent scattering makes the characteristic decay length shorter. Account for this scattering changes the quantity κ_ω as $\kappa_\omega \Rightarrow \kappa_\omega = 1 + 2|\omega|\tau + \lambda_\perp + (4/9)\lambda_{so}$ (see Sec. V). As follows from Eqs. (17) and (18), the SR components, $\hat{S}_\pm$, arise in the case of a homogenous magnetization when $H_\omega = 0$. The LRTC appears only in the presence of a nonhomogeneous magnetization, for example, in the presence of a DW when $H_\omega \sim (h\tau)(\bar{w}/l) \neq 0$.

Equations (17) and (18) are the main results of the paper. They determine the spatial dependence of the short, $\hat{s}_\pm$, and long, $\hat{s}_1$, range amplitudes of the condensate. Note that the amplitudes of the singlet, $\hat{S}_3(k)$, and short-range triplet, $\hat{S}_0(k)$, components are expressed through $\hat{S}_\pm(k)$ in a simple way,

$$\hat{S}_{0,3}(k) = [\hat{S}_+(k) \pm \hat{S}_-(k)]/2, \tag{19}$$

Although Eqs. (17) and (18) completely determine the solutions of Eqs. (15) and (16), the explicit form of the solutions is still to be obtained from the inverse Fourier transform. Unfortunately, the latter can be presented in an analytical form only in some limiting cases. In Sec. III we will

analyze the spatial dependence of the amplitudes $\hat{s}_{0,1,3}(x)$.

III. SPATIAL DEPENDENCE OF THE CONDENSATE WAVE FUNCTIONS

Using Eqs. (17) and (18) one can obtain the spatial dependence of the amplitudes $\hat{s}_{0,1,3}(x)$ describing the penetration of the odd triplet condensate into the ferromagnet. The corresponding expressions are to be found by calculating the inverse Fourier transform of Eqs. (17) and (18).

The form of the expressions for $\hat{S}_\pm(k)$ and $\hat{S}_1(k)$ indicates that the spatial dependence of the amplitudes $\hat{s}_{0,1,3}(x)$ is determined by zeros of the functions $M_\omega(k,\mu)$ and $M_{h\pm}(k,\mu)$ as well as of the functions $(1-\kappa_\omega\langle M_\omega^{-1}(k,\mu)\rangle)$ and $(1-\kappa_{h\pm}\langle M_{h\pm}^{-1}(k,\mu)\rangle)$. Although the decay length of the amplitudes $\hat{s}_{0,3}(k)$ depends on the exchange energy h, the decay length of the amplitude $\hat{s}_1(k)$ does not.

The LRTC in the Fourier representation, $\hat{S}_1(k)$, is expressed through the short-range singlet component $\hat{s}_3(0)$ at $x=0$, and in its turn, the matrix $\hat{S}_\pm(k)$ depends on the amplitude of the singlet component in S, $\hat{f}_S$, and on the LRTC $\hat{s}_1(0)$ at $x=0$. We suppose that the influence of the LRTC on the short-range amplitude $\hat{S}_\pm(k)$ is weak; that is, the condition

$$H|s_1(0)| \ll b_\mu f_S \tag{20}$$

is satisfied.

The matrices $\hat{S}_\pm(k)$ and $\hat{S}_1(k)$ may be found from Eqs. (17) and (18), respectively. Then, performing the integration over k, one can find $s_{0,3}(0)=\int(dk/2\pi)s_{0,3}(k)$ and $s_1(0)$. However, the expressions for these matrices are too cumbersome even if condition (20) is fulfilled. One can further simplify these expressions considering the limits of large and small products $h\tau$, i.e., considering the case of a strong or weak ferromagnet. We also will assume that the condition

$$T\tau \ll 1 \tag{21}$$

is fulfilled because it corresponds to experimental situations. The corresponding formulas can easily be obtained also in the opposite limit.

First, we consider the limit of a weak ferromagnet when the inequality (a) $\{h,T\} \ll \tau^{-1}$ is fulfilled (the diffusive case or the case of a weak ferromagnet). In this case, the main contribution comes from small k ($k \ll 1$) (Ref. 47) and we obtain: $1-\kappa_{h\pm}\langle M_{h\pm}^{-1}(k,\mu)\rangle \approx (k^2+K_h^2)/3$ and $1-\kappa_\omega\langle M_\omega^{-1}(k,\mu)\rangle \approx (k^2+K_\omega^2)/3$, where $K_h^2=6(|\omega| \mp ih_\omega)\tau=l\sqrt{(2|\omega| \mp ih_\omega)/D}$ and $K_\omega^2=3(\kappa_\omega-1)$. Calculating the residue of the pole at $k=iK_h$ in Eq. (17), we find for the amplitudes of the SR components

$$\hat{s}_\pm(x) = \pm 3(b_\mu/K_{h\pm})\hat{f}_S \exp(-x/\xi_{h\pm}), \tag{22}$$

where $\xi_{h\pm}=\sqrt{D/[2(|\omega| \mp ih_\omega)]}$ is the characteristic length over which the short-range components (singlet and triplet ones with zero projection S_z on the z axis) penetrate the ferromagnet.

The LRTC can be found from Eq. (18) calculating the residue and we obtain

$$\hat{s}_1(x) = -18H_\omega\langle b_\mu\rangle(\xi_{LR}/l)\mathrm{Re}\frac{1}{K_h}(\hat{\tau}_3 \cdot \hat{f}_S)\exp(-x/\xi_{LR}), \tag{23}$$

with $\xi_{LR}=\sqrt{D/(2|\omega|\tau+\lambda_\perp+(4/9)\lambda_{so})}$ and $K_h=\sqrt{6(|\omega|-ih_\omega)\tau}$.

Equations (22) and (23) describe the spatial dependence of the short- and long-range components of the condensate. The SR component $\hat{s}_\pm(x) \sim \hat{\tau}_2 \exp(-x/\xi_{h\pm})$ decays over a short length, $\xi_{h\pm}$, and experiences oscillations.[11,12,48] The LRTC $\hat{s}_1(x) \sim \hat{\tau}_1 \exp(-x/\xi_\omega)$ decays without oscillations over a long distance ξ_{LR}.[10–12] The amplitude of the singlet component $\hat{s}_3(0)=s_3\hat{\tau}_2$ at $x=0$ equals

$$\hat{s}_3(0) = 3b_\mu \,\mathrm{Re}\frac{1}{K_h}\hat{f}_S. \tag{24}$$

Thus, the ratio of the LRTC $\hat{s}_1(0)=s_1\hat{\tau}_1$ to the singlet component $s_3(0)$ takes the form

$$r = \left|\frac{s_1(0)}{s_3(0)}\right| = 2\frac{\xi_{LR}}{\xi_h^2}\bar{w}. \tag{25}$$

This ratio may be both larger and smaller than 1. The amplitude of the LRTC increases with increasing the exchange energy h.

Condition (20) can be rewritten in this limit as

$$\left(\frac{\bar{w}}{l}\right) \ll \frac{1}{3\sqrt{6}}\frac{\xi_{LR}}{(h\tau)^{3/2}l}. \tag{26}$$

If the spin-coupling constant λ_{so} is larger than the product $|\omega|\tau$, this inequality can be written as $h\tau \ll (1/18)\lambda_{so}$.

Now we consider the limit of the large exchange energy h: (b) $T \ll \tau^{-1} \ll h$ (the case of a strong ferromagnet). In this case the quantity $\kappa_{h\pm}\langle M_{h\pm}^{-1}(k,\mu)\rangle$ is small because $|\kappa_{h\pm}| \gg 1$. Therefore, the main contribution to $\hat{s}_\pm(x)$ is due to the second term in the figure brackets in Eqs. (17) and one has to calculate the residue of the pole of the functions $(M_{h\pm}(k,\omega))^{-1}$. The formula for $\hat{s}_1(x)$ is obtained as before.

As a result, we find

$$\hat{s}_\pm(x) = \pm (b_\mu/|\mu|)\hat{f}_S \exp(-\kappa_{h\pm}x/l) \tag{27}$$

and

$$\hat{s}_1(x) = -6\langle b_\mu/|\mu|\rangle H_\omega(\xi_{LR}/l)(\hat{\tau}_3 \cdot \hat{f}_S)\exp(-x/\xi_{LR}) \tag{28}$$

The SR components $\hat{s}_\pm(x)$ oscillate with the period $\pi v/h$ and decrease in the ferromagnet over the mean free path l as has been obtained earlier in this limit.[47,49,50] The LRTC decreases in a monotonic way over the length ξ_{LR}. The ratio of the amplitude of the LRTC to the short-range singlet component at $x=0$ is equal to

$$r = 6\left(\frac{\bar{w}}{v/h}\right)\frac{\xi_{LR}}{l}, \tag{29}$$

and condition (20) is fulfilled provided the inequality

$$\bar{w} \ll \frac{v}{2h}\sqrt{\frac{l}{3\xi_{LR}}} \qquad (30)$$

is satisfied. Combining Eqs. (29) and (30), one obtains that the ratio of the LRTC and singlet component at the SF interface satisfies the condition

$$r < \sqrt{3\frac{\xi_{LR}}{l}}. \qquad (31)$$

If the spin-dependent scattering can be neglected, this inequality can be written as $r < (2T\tau)^{-1/4}$. This means that for $T \approx 4K$ and $\tau \approx 10^{-14}$ s, the ratio r should be $r \lesssim 6$; that is, the amplitude of the LRTC at the SF interface may be comparable with or even larger than the singlet component. We see that at a given width of the DW w, the amplitude of the LRTC increases with increasing exchange field h, whereas the amplitude of the singlet component $s_3(0)$ decreases and reaches an asymptotic value $\sim b_\mu$ at $h\tau \gg 1$. The maximum value of the LRTC at $h\tau \gg 1$ is of the order of $\sim b_\mu \sqrt{\xi_{LR}/l}$. The upper limit on h is imposed by the condition: $w < v/h$, i.e., max $h \approx v/w$. In the both cases of small and large product $h\tau$, the amplitude of the LRTC is proportional to the width of the DW turning to zero at $w=0$.

IV. JOSEPHSON EFFECT

In this section we consider the dc Josephson effect in an SFS junction with narrow DWs located at the left and right interfaces. We assume that the distance between the superconductors is larger than the correlation length $\xi_{SN} = \sqrt{D/\Delta}$ in the absence of the exchange field. In this case, the Josephson coupling is caused only by the LRTC and the overlap of the LRTC created by each interface is weak. Then, in order to calculate the Josephson critical current, one can represent the amplitude of the LRTC in the form

$$\check{s}(x) = \check{s}_L(x) + \check{s}_R(x), \qquad (32)$$

where $\check{s}_{L,R}(x)$ are the amplitudes of the LRTC created by the left (right) interfaces. These matrices are equal to

$$\check{s}_L(x) = -\hat{\sigma}_1 \otimes \hat{\tau}_3 \cdot \hat{f}_S \frac{\xi_{LR}}{l} B_L \exp(-x/\xi_\omega), \qquad (33)$$

$$\check{s}_R(x) = -\hat{\sigma}_1 \otimes \hat{\tau}_3 \cdot \hat{S} \cdot \hat{f}_S \cdot \hat{S}^\dagger \frac{\xi_{LR}}{l} B_R \exp(-(L-x)/\xi_\omega), \qquad (34)$$

where the coefficients $b_{L,R}$ equal $B_{L,R} = 18 H_{\omega L,R}\langle b_\mu \rangle_{L,R} \mathrm{Re}(1/K_h)$ if $h\tau \ll 1$ and $B_{L,R} = 6 H_{\omega L,R}\langle b_\mu/\mu \rangle_{L,R}$ if $h\tau \gg 1$.

With the help of the matrix $\hat{S} = \cos(\varphi/2) + i\hat{\tau}_3 \sin(\varphi/2)$, we take into account the phase difference φ between the superconductors S (the phase of the left S is set to be equal to zero). In order to compare the magnitude of the Josephson critical current in the considered case of the SFS junction with the one for an SNS junction, we write down here also the amplitude of the singlet component for the SNS junction expressed in terms of the same quantities. We can obtain it

from Eq. (22) simply setting $h=0$ and the corresponding expressions take the form

$$\check{s}_L(x) = 3\hat{f}_S \otimes \hat{\sigma}_3 \langle b_\mu \rangle \frac{\xi_{LR}}{l} \exp(-x/\xi_\omega), \qquad (35)$$

$$\check{s}_R(x) = 3\hat{S}\cdot\hat{f}_S \otimes \hat{\sigma}_3 \cdot \hat{S}^\dagger \langle b_\mu \rangle \frac{\xi_{LR}}{l} \exp(-(L-x)/\xi_\omega), \qquad (36)$$

with $\hat{f}_S = f_S \hat{\tau}_2$ and $f_S = \Delta/\sqrt{\omega^2 + \Delta^2}$.

The current through the SFS (or SNS) junction in the limit $T\tau \ll 1$ is given by the expression

$$I = \frac{1}{16}\mathcal{S}\sigma(4\pi Ti)\mathrm{Tr}(\hat{\sigma}_0 \otimes \hat{\tau}_3)\sum_\omega \{\check{s}(x)\,\partial\check{s}(x)/\partial x\}, \qquad (37)$$

where $\mathcal{S}$ is the cross-section area of the junction and the summation is performed over the fermionic Matsubara frequencies.

Substituting the function $\check{s}$ from Eqs. (32)–(34) into this expression, we obtain for the case of identical interfaces

$$I_J = \frac{9}{8}\mathcal{S}\sigma(4\pi Ti)\langle b_\mu \rangle^2 \mathrm{Tr}\,\hat{\tau}_3 \sum_\omega \{\hat{f}_S \cdot \hat{S} \cdot \hat{f}_S \cdot \hat{S}^\dagger$$

$$- \hat{S}\cdot\hat{f}_S \cdot \hat{S}^\dagger \cdot \hat{f}_S\} \frac{\xi_{LR}}{l}\exp(-L/\xi_\omega). \qquad (38)$$

Calculating the trace in Eq. (38), we find

$$I_J = I_{c(SNS)}\sin\varphi, \quad I_{c(SNS)}3\sqrt{3/2}\mathcal{S}\sigma(4\pi T)$$

$$\times \langle b_\mu \rangle^2 \sum_{\omega=0} f_S^2(\omega)\frac{\exp(-L/\xi_{LR})}{l\sqrt{\omega\tau}}. \qquad (39)$$

A similar formula for I_J can be obtained for the SFS junction with the use of the LRTC $\check{s}$ given by Eqs. (33) and (34). We write down the expression for the critical currents caused by the LRTC,

$$I_{c(SFS)} = -H_{\omega L}H_{\omega R}6\sqrt{6}\mathcal{S}\sigma(4\pi T)\langle b_\mu \rangle_L \langle b_\mu \rangle_R \sum_{\omega=0} f_S^2 \frac{\exp(-L/\xi_\omega)}{l\sqrt{\omega\tau}},$$

$$h\tau \gg 1, \qquad (40)$$

where ξ_ω is given, as before, by Eq. (2).

The sign opposite to the critical current $I_{c(SNS)}$ in Eq. (40) arises because the product $(\hat{f}_S \cdot \hat{S} \cdot \hat{f}_S \cdot \hat{S}^\dagger)$ in Eq. (38) is replaced in the case of SFS junction by the product $(\hat{\tau}_3 \cdot \hat{f}_S \cdot \hat{S} \cdot \hat{\tau}_3 \cdot \hat{f}_S \cdot \hat{S}^\dagger)$. If the interfaces and domain walls in SFS are identical ($H_L = H_R \equiv H$), we get

$$I_{c(SFS)} = -4H^2 I_{c(SNS)}, \qquad (41)$$

where $H = (h\tau)(w/l)\langle\sin\alpha\rangle_w$.

According to inequality (20) the quantity $4H^2$ must satisfy the condition $4H^2 < \sqrt{\omega\tau} < 1$. This means that the critical current in SFS junctions with a narrow domain wall is smaller than the critical current $I_{c(SNS)}$ of the SNS junction. However, it can become comparable with the latter provided the parameter H is of the order of 1, which is possible for strong ferromagnets ($h\tau \gg 1$). In the case of different orientations of

magnetization on the right and left domain walls, i.e., if the product $\langle \sin \alpha \rangle_{wL} \langle \sin \alpha \rangle_{wR}$ is negative, the critical current $I_{c(\mathrm{SFS})}$ has the same sign as $I_{c(\mathrm{SNS})}$. This result is in accordance with the results in Ref. 37, where the sign of the critical current was shown to be sensitive to a so-called chirality depending on whether the magnetization vector **M** rotated or oscillated when going from one interface to the other. The negative sign of the critical current in the SFS junction with a half metal was obtained also in Ref. 19.

V. SPIN-DEPENDENT SCATTERING IN A DIFFUSIVE SF BILAYER

In this section we consider for completeness the diffusive limit assuming that the mean free path l is shorter than the "magnetic" length ξ_h, Eq. (1). However, in contrast to Ref. 10 the width of the DW, w, is supposed to be shorter than the length ξ_h. Then, in order to find the LRTC, we can use the same method as in Secs. II and IV. We also take into account the spin-dependent scattering that strongly affects the penetration length of the LRTC. In the case of a diffusive SF bilayer considered here, one can use the Usadel equation which in the F layer has the form

$$D \, \partial \, (\breve{g} \, \partial \breve{g}/\partial x^2) - [\omega \hat{\tau}_3 - ih\hat{\tau}_3 \otimes \hat{\sigma}_3 \cos \alpha(x), \breve{g}]$$
$$+ ih[\hat{\tau}_0 \otimes \hat{\sigma}_2 \sin \alpha(x), \breve{g}] = \breve{I}_m/\tau, \quad (42)$$

where $\breve{g}$ is a 4×4 matrix Green's function in the ferromagnetic region that does not depend in the diffusive limit on the momentum orientation, and $D = vl/3$ is the diffusion coefficient. The matrix on the right-hand side is the spin-dependent collision term,

$$2\breve{I}_m = \{\breve{m}\langle \breve{g} \rangle \breve{m}\breve{g} - \breve{g}\breve{m}\langle \breve{g} \rangle \breve{m} + \langle \breve{g} \rangle_{so}\breve{g} - \breve{g}\langle \breve{g} \rangle_{so}\}_\varphi. \quad (43)$$

with $\breve{m} = 1 + \lambda_z \breve{n}_z + \lambda_\perp \breve{n}_\perp$, $\breve{n}_z = \hat{\tau}_3 \otimes \hat{\sigma}_3$, and $\breve{n}_\perp = \hat{\tau}_0 \otimes (\hat{\sigma}_1 \cos \varphi + \hat{\sigma}_2 \sin \varphi)$. The subscript φ means the averaging over the azimuthal angle φ.

The last two terms in Eq. (43) stand for the spin-orbit scattering,

$$\langle \breve{g} \rangle_{so} = (\lambda_{so}/4\pi) \int d\Omega' e_i' e_k' (\mathbf{S} \times \mathbf{e})_i \breve{g} (\mathbf{S} \times \mathbf{e})_k. \quad (44)$$

with $\mathbf{S} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\tau}_3 \otimes \hat{\sigma}_3)$. The coefficients $\lambda_{z,\perp}$ and λ_{so} are expressed in terms of spatial fluctuations of the magnetic moments of impurities (see Refs. 11 and 51). For example, the most important coefficient, λ_{so}, related to the spin-orbit interaction is equal to $\lambda_{so} = \tau/\tau_{so}$, $\tau^{-1} = 2\pi\nu N_{\mathrm{imp}} u_{\mathrm{imp}}^2$, and $\tau_{so}^{-1} = 2\pi\nu N_{\mathrm{imp}} \int d\Omega/4\pi u_{so}^2 \sin^2 \theta$, where ν is the density of states at the Fermi level, which is assumed to be the same for the spin-up and spin-down orientations in the quasiclassical approximation, N_{imp} is the impurity concentration, and u_{imp} and u_{so} is the potential of impurities and spin-orbit interaction, respectively. These coefficients are related to the quantities used in Ref. 52, $\Gamma_{x,z}$ and Γ_{so}, in the following way: $2\tau\Gamma_{x,z} = \lambda_{\perp,z}$ and $9\tau\Gamma_{so} = \lambda_{so}$.

We employ the boundary condition at the SF interface in the form presented in Ref. 53,

$$\breve{g} \, \partial \breve{g}/\partial x|_{x=0} = (2\gamma_F)^{-1}[\breve{g}_S, \breve{g}], \quad (45)$$

where $\gamma_F = R_B \sigma_F = l/b_{av}$, $b_{av} = c_1 T_{av}$, T_{av} is an effective transmission coefficient averaged over angles, and c_1 is a numerical factor of the order of 1.[46,53] Integrating Eq. (42) over the width of the DW, we obtain an effective boundary condition for the Usadel equation,

$$\breve{g} \, \partial \breve{g}/\partial x|_{x=w} = (2\gamma_F)^{-1}[\breve{g}_S, \breve{g}] - iK_D[\hat{\tau}_0 \otimes \hat{\sigma}_2, \breve{g}], \quad (46)$$

where $K_D = (hw/D)\langle \sin \alpha(x) \rangle_w \equiv h\bar{w}/D$.

We assume again that the proximity effect is weak so that the matrix $\breve{g}$ can be represented in the form of Eq. (5). Then, we linearize Eqs. (42) and (46) and arrive at the equation for $\breve{f}$ in the region outside the domain wall ($x > w$),

$$\partial^2 \breve{f}/\partial x^2 - 2\varkappa_\omega^2 \breve{f} + i\varkappa_h^2[\hat{\sigma}_3, \breve{f}]_+ = \varkappa_{\mathrm{non}}^2 \delta \breve{I}_m, \quad (47)$$

where $\varkappa_{\mathrm{non}} = 1/\sqrt{D\tau}$ is a wave vector related to a nonmagnetic scattering, $\delta \breve{I}_m = \delta \breve{I}_{sp} + \delta \breve{I}_{so}$ and

$$\delta \breve{I}_{sp} = \{\lambda_z^2(\breve{f} + \hat{\sigma}_3 \otimes \breve{f} \otimes \hat{\sigma}_3)$$
$$+ \lambda_\perp^2[\breve{f} - (\hat{\sigma}_1 \otimes \breve{f} \otimes \hat{\sigma}_1 + \hat{\sigma}_2 \otimes \breve{f} \otimes \hat{\sigma}_2)/2]\}, \quad (48)$$

$$\delta \breve{I}_{so} = (\lambda_{so}/3)\{\breve{f} + (\hat{\sigma}_1 \otimes \breve{f} \otimes \hat{\sigma}_1 + \hat{\sigma}_2 \otimes \breve{f} \otimes \hat{\sigma}_2$$
$$- \hat{\sigma}_3 \otimes \breve{f} \otimes \hat{\sigma}_3)/3\}. \quad (49)$$

The effective boundary conditions are obtained as before and have the form

$$\partial \breve{f}/\partial x|_{x=0} = (1/\gamma_F)(|g_S|\breve{f} - \breve{f}_S) - iK_D\hat{\tau}_3 \otimes [\hat{\sigma}_2, \breve{f}],$$
$$\partial \breve{f}/\partial x|_{x=d} = 0. \quad (50)$$

Here $\varkappa_\omega^2 = |\omega|/D$, $\varkappa_h^2 = -h_\omega/D$, $K_D = h_\omega w\langle \sin \alpha(x) \rangle_w/D$, and $g_S = \omega/\sqrt{\omega^2 + \Delta^2}$. We again seek for a solution in the form

$$\breve{f}(x) = \hat{\tau}_2 \otimes [\hat{\sigma}_3 f_3(x) + \hat{\sigma}_0 f_0(x)] + \hat{\tau}_1 \otimes \hat{\sigma}_1 f_1(x), \quad (51)$$

where $f_3(x)$ is the amplitude of the singlet component and $f_{0,1}(x)$ are the amplitudes of the short-range $S_z = 0$ and long-range $|S_z| = 1$ triplet components, respectively.

In this section we consider an SF bilayer of a finite width having in mind to calculate the DOS variation at the outer surface of the F layer. In order to satisfy the second boundary condition at $x = d$, Eq. (50), we represent the solution in the form

$$f_{0,3}(x) = \mp C_{0,3+} \cosh[\varkappa_+(x-d)] + C_{0,3-} \cosh[\varkappa_-(x-d)], \quad (52)$$

$$f_1(x) = C_1 \cosh[\varkappa_1(x-d)], \quad (53)$$

with the decay lengths determined by $\varkappa_\pm$ and $\varkappa_1$.

Substituting Eqs. (51)–(53) into Eq. (47), we obtain a system of equations for the coefficients $C_{0,3}$ and C_1,

$$C_0(\varkappa^2 - 2\varkappa_\omega^2 - K_0^2) + 2i\varkappa_h^2 C_3 = 0, \quad (54)$$

$$C_3(\varkappa^2 - 2\varkappa_\omega^2 - K_3^2) + 2i\varkappa_h^2 C_0 = 0, \quad (55)$$

$$C_1(\varkappa^2 - 2\varkappa_\omega^2 - K_1^2) = 0, \tag{56}$$

where $K_0^2 = 2\varkappa_{\text{non}}^2(\lambda_z + (2/9)\lambda_{so})$, $K_3^2 = 2\varkappa_{\text{non}}^2(\lambda_z + \lambda_\perp)$, and $K_1^2 = \varkappa_{\text{non}}^2(\lambda_\perp + (4/9)\lambda_{so})$.

We see that the wave vector characterizing the decay of the singlet component does not depend on the spin-orbit scattering as it should be. Note that the influence of the spin-orbit scattering on the SR components has been considered for the first time in Ref. 54. The eigenvalue of Eq. (56) $\varkappa_1^2$ equals

$$\varkappa_1^2 = 2\varkappa_\omega^2 + K_1^2. \tag{57}$$

The eigenvalues $\varkappa_\pm^2$ that determine the relation between the coefficients $C_{0,3}$ are found from Eqs. (54) and (55). They are the roots of the equation

$$(\varkappa^2 - 2\varkappa_\omega^2 - K_0^2)(\varkappa^2 - 2\varkappa_\omega^2 - K_3^2) + 4\varkappa_h^4 = 0. \tag{58}$$

As follows from Eq. (58), both the eigenvalues are real provided the condition

$$4\varkappa_h^2 < |K_0^2 - K_3^2| \tag{59}$$

is fulfilled. In this case there are no oscillations in the condensate functions and, therefore, no oscillations of observable quantities.

In the limit

$$\varkappa_h^2 \gg 2\varkappa_\omega^2, K_{0,3,1}^2, \tag{60}$$

the eigenvalues equal

$$\varkappa_\pm^2 \approx \pm 2i\varkappa_h^2 + 2\varkappa_\omega^2 + (K_0^2 + K_3^2)/2. \tag{61}$$

The coefficients $C_{0,3}$ and C_1 are found from Eqs. (54) and (55) and the first boundary condition, Eq. (50). Under condition (60) they are equal to

$$C_{3\pm} \approx \mp C_{0\pm} \approx \frac{d}{\gamma_F} \frac{1}{2A_\pm} f_s, \quad C_1 \approx \frac{K_D d^2}{\gamma_F} \frac{1}{A_1} f_s \left(\frac{1}{\widetilde{A}_+} + \frac{1}{\widetilde{A}_-} \right), \tag{62}$$

where $f_s = \Delta/\sqrt{\omega^2 + \Delta^2}$, $A_\pm = \theta_\pm \sinh \theta_\pm + (d/\gamma_F)g_S \cosh \theta_\pm$, $A_1 = \theta_1 \sinh \theta_1 + (d/\gamma_F)g_S \cosh \theta_1$, $\theta_\pm = \varkappa_\pm d$, and $\widetilde{A}_\pm = \theta_\pm \tanh \theta_\pm + (d/\gamma_F)g_S$. Again, we neglected the influence of the LRTC on the SR components. This is justified provided the condition

$$2K_D d C_1 \ll (d/\gamma_F) f_S \tag{63}$$

is fulfilled.

As follows from Eq. (57), the spin-dependent scattering can essentially reduce the penetration depth for the LRTC. This holds also for the cases considered in Secs. II–IV. Equations (52)–(62) describe the spatial dependence of the SR, $f_{3,0}(x)$, and LR, $f_1(x)$, components. In particular, at the outer boundary of the ferromagnet we have $f_{3,0}(d) = \pm C_{3+} + C_{3-}$ and $f_1(d) = C_1$. This means that the short-range components oscillate and decay over a distance of the order of ξ_h: $f_{3,0}(d) \sim \exp(-(1+i)d/\xi_h)$ at $d/\xi_h \gg 1$, whereas the LRTC, $f_1(d)$, decays in a monotonous way over a longer distance $\sim \varkappa_1^{-1}$. The ratio of the LRTC and singlet component at the interface is equal to

$$r = \frac{|f_1(0)|}{|f_3(0)|} = \frac{2w(h/D)\langle \sin \alpha \rangle_w}{\varkappa_1 \tanh \theta_1 + \gamma_F^{-1}|g_S|}. \tag{64}$$

This quantity may be both larger or less than unity. In Sec. VI we calculate the DOS by using the results for $f_{0,3,1}(x)$ obtained here.

VI. DOS IN A DIFFUSIVE SF BILAYER

The DOS variation, $\delta\nu = \nu - 1$, in the ferromagnetic film caused by proximity effect in SF bilayers was measured in a number of works.[44,55,56] In particular, the inversion of $\delta\nu$ with increasing the thickness of the F layer d was observed. This effect has been explained theoretically in terms of the SR component oscillations in space.[56–59]

An interesting, although small, effect has been observed in a recent work.[44] The authors measured the DOS at the outer surface of the F in an SF system for various thicknesses of the ferromagnet d. They identified two small peaks in the variation of the DOS. One of these peaks corresponded to the energy gap Δ in the superconductor, whereas the other one corresponded to a smaller energy. The first peak inverted with increasing d but the sign of the second peak remained unchanged.

SanGiorgio et al.[44] suggested an explanation of this effect assuming that the second peak is due to a contribution of the LRTC. At the same time, this peak cannot be a result of a long-range penetration of the LRTC into the ferromagnet but is rather due to a different (monotonous) dependence on the thickness d. In this section, we represent the contributions of the SR and LR components to the DOS in the ferromagnetic layer using Eqs. (52), (53), and (62). We demonstrate that the contribution due to the SR components, as it was shown earlier, changes the sign with increasing d, while the contribution due to the LR component does not. We are not going to make a detailed comparison with the experimental results because not all necessary data are available. For example, nothing is known about the domain structure in the F layer.

In calculating the DOS, we use parameters close to estimates presented in Ref. 44: $(\gamma_F d)^{-1} = d/(\xi_s \gamma_B) \approx 0.3$ for $d = 4$ nm, $\gamma_B = 0.5$, and $\xi_s = \sqrt{D/2\Delta} = 23$ nm. In the considered case of a weak proximity effect, the correction to the DOS, $\delta\nu(\epsilon)$, at boundary $x = d$ is equal to

$$\delta\nu(\epsilon) = -(1/2)\text{Re}[f_3^2(d) + f_0^2(d) + f_1^2(d)]_{\omega = -i\epsilon}, \tag{65}$$

where the condensate functions $f_{3,0,1}^2(d)$ are determined by Eqs. (52), (53), and (62).

In Figs. 2 and 3 we plot the contributions to the DOS from the singlet, f_3, SR triplet, f_0, and LR triplet, f_1, components as a function of energy ϵ for two different thicknesses of the ferromagnetic layer. To be more precise, in Figs. 2(a) and 3(a) the corrections $\delta\nu_{SR} = -(1/2)\text{Re}[f_3^2(d) + f_0^2(d)]$ due to the SR components are plotted, whereas in Figs. 2(b) and 3(b) we show the dependence of $\delta\widetilde{\nu}_{LR} = a^{-1}\delta\nu_{LR}$ versus energy, where $\delta\nu_{LR} = -(1/2)\text{Re}(f_1^2)$ and $a = (\overline{w}/d)\theta_h^2$. That is, in order to get the actual contribution to the DOS due to the LR component, the magnitudes shown in Figs. 2(b) and 3(b) should be multiplied by a.

We see that the corrections to the DOS due to the SR components change sign with increasing d, whereas the sign
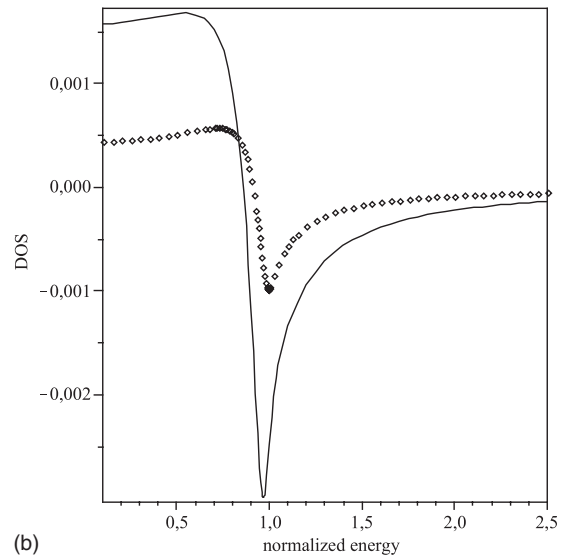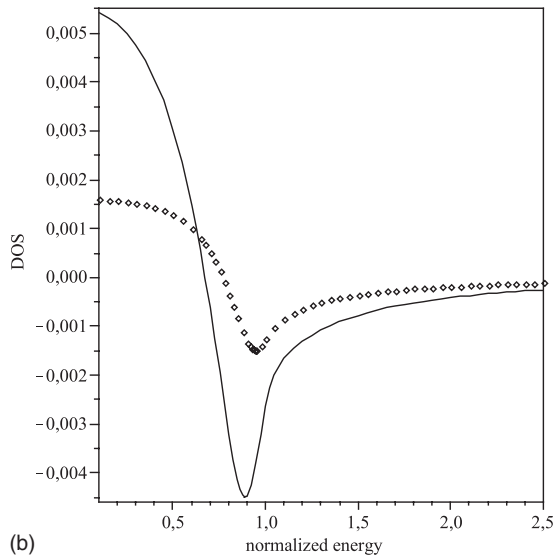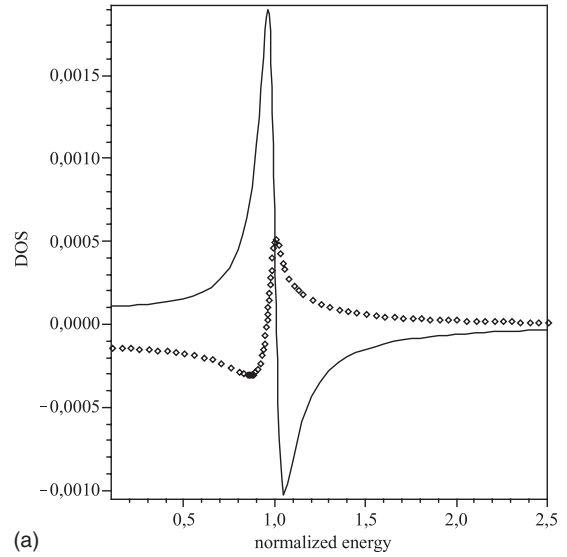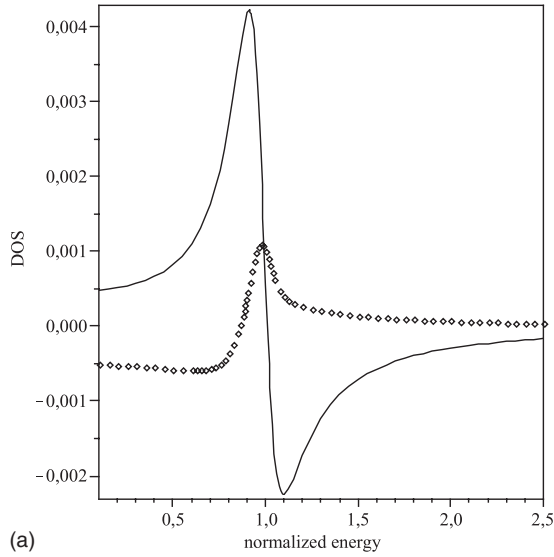
FIG. 2. DOS variation $\delta\nu(\epsilon)$ at the outer surface of the F layer vs the normalized energy ϵ/Δ plotted on the basis of Eqs. (52) and (53). The contributions of the SR components, $f_{0,3}$, are shown in (a) and the contributions of the LRTC, f_1, is shown in (b). The following values of parameters are used: $\theta_h \equiv d/\xi_h = 1.5$ (solid lines) and $\theta_h = 1.8$ (dotted lines); $\Gamma/\Delta = 0.1$, $d/\gamma_F = 0.4$, and $K_1 = 0.6\varkappa_h$. The parameters $\varkappa_\epsilon$ and K_3 are assumed to be much smaller than $\varkappa_h$.

FIG. 3. The same graphs as in Fig. 2 with parameters: $\theta_h \equiv d/\xi_h = 1.5$ (solid lines) and $\theta_h = 1.8$ (dotted lines); $\Gamma/\Delta = 0.05$, $d/\gamma_F = 0.2$, and $K_1 = 0.6\varkappa_h$. The parameters $\varkappa_\epsilon$ and K_3 are assumed to be much smaller than $\varkappa_h$.

with that of the SR components if the width of the DW, w, is comparable with ξ_h.

of the correction due to the LRTC remains unchanged. It is also worth mentioning that, strictly speaking, singularities in the SR and LR components correspond to different energies. If condition (60) is fulfilled, only the function $f_S(\epsilon) = i\Delta/\sqrt{(\epsilon+i\Gamma)^2-\Delta^2}$ depends on the energy ϵ, and therefore, the position of singularities is determined only by the energy gap Δ and damping constant Γ.

On the other hand, the first term in the expression for $A_1 = \theta_1 \sinh \theta_1 + (d/\gamma_F)g_S(\epsilon)\cosh \theta_1$ may be comparable with the second one that also depends on the energy ϵ. The account for the second term leads to a decrease in a characteristic energy that determines the position of the singularity. One can see that the contribution of the LRTC is comparable

VII. CONCLUSIONS

We have considered the long-range triplet component in an SF bilayer arising due to an nonhomogeneous magnetization in the F layer (for example, due to a domain wall) located in the vicinity of the SF interface. Unlike Refs. 10 and 11 where the width of the DW, w, was assumed to be larger than the mean free path, we have calculated in the present paper the amplitudes of the LR as well as of the SR components for the case of a narrow DW. In fact, our model may be considered as a microscopic model of a spin-active SF interface usually described by introducing phenomenological parameters.

Assuming that the proximity effect is weak (this corresponds to experimental data), we have obtained analytical formulas for the amplitudes of the LR and SR components in a wide range of parameters. The amplitudes of the SR components decrease with increasing exchange energy h and become constant at $h\tau \gg 1$. The amplitude of the LRTC essentially depends on the parameter $h\tau$ and increases with increasing h. The maximum value of the amplitude of the LRTC in our approach is determined by the condition $h < (v/w)$.

We have calculated the critical Josephson current I_c in a SFS junction where the Josephson coupling is due to the LRTC. The current I_c is negative if the rotation of the magnetization vector **M** in DWs at each SF interface occurs in one direction (positive chirality) and is positive if the rotation of **M** occurs in different direction (negative chirality). We have also found the DOS at the outer surface of the F layer in an SF structure in the presence of a DW at the SF interface. It has been shown that contributions to the DOS from the SR and LR components have singularities at an energy $\sim\Delta$. Whereas the singularity due to the SR components changes sign with increasing the thickness of the F layer, d, the singularity due to the LR component does not. The change in sign occurs at $d \approx (\pi/2)\xi_h$. Note also that the contribution of the LRTC to the DOS is of the same order as the one of the SR components provided the width of the DW is comparable with the length ξ_h.

We considered the case of the DW parallel to the SF interface. However, this fact is not crucial: the LRTC created by DWs perpendicular to the SF interface may be of the same order as the LRTC induced by the DW parallel to the SF interface. The amplitude of the LRTC for the case of the Néel DWs perpendicular to the SF interface has been calculated in Ref. [42]. One can show that similar results can be obtained for the case of the Bloch DWs perpendicular to the SF interface.[60] In order to carry out a more detailed comparison with experiments, more data are required. In particular, one has to know the parameters of the magnetic structure of the F film.

ACKNOWLEDGMENTS

---

[1] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).

[2] J. R. Schrieffer, *Superconductivity* (Benjamin, New York, 1964).

[3] C. C. Tsuei and J. R. Kirtley, Rev. Mod. Phys. **72**, 969 (2000).

[4] V. P. Mineev and K. V. Samokhin, *Introduction to Unconventional Superconductivity* (Gordon and Breach, Amsterdam, 1999).

[5] A. P. Mackenzie and Y. Maeno, Rev. Mod. Phys. **75**, 657 (2003).

[6] I. Eremin, D. Manske, S. G. Ovchinnikov, and J. F. Annett, Ann. Phys. (N.Y.) **13**, 149 (2004).

[7] V. L. Berezinskii, JETP Lett. **20**, 287 (1975).

[8] A. J. Leggett, Rev. Mod. Phys. **47**, 331 (1975).

[9] D. Vollhardt and P. Wölfle, *The Superfluid Phases of He³* (Taylor & Francis, London, 1990).

[10] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Phys. Rev. Lett. **86**, 4096 (2001).

[11] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Rev. Mod. Phys. **77**, 1321 (2005).

[12] A. Buzdin, Rev. Mod. Phys. **77**, 935 (2005).

[13] S. N. Artemenko, A. F. Volkov, and A. V. Zaitsev, Solid State Commun. **30**, 771 (1979).

[14] V. N. Gubankov and N. M. Margolin, JETP Lett. **29**, 673 (1979).

[15] Y. V. Nazarov and T. H. Stoof, Phys. Rev. Lett. **76**, 823 (1996).

[16] A. F. Volkov, N. Allsopp, and C. J. Lambert, J. Phys.: Condens. Matter **8**, 45 (1996).

[17] P. Charlat, H. Courtois, P. Gandit, D. Mailly, A. F. Volkov, and B. Pannetier, Phys. Rev. Lett. **77**, 4950 (1996).

[18] A. Kadigrobov, R. I. Shekhter, and M. Jonson, Europhys. Lett. **54**, 394 (2001).

[19] M. Eschrig, J. Kopu, J. C. Cuevas, and G. Schön, Phys. Rev. Lett. **90**, 137003 (2003).

[20] Y. Asano, Y. Sawa, Y. Tanaka, and Alexander A. Golubov, Phys. Rev. B **76**, 224525 (2007).

[21] A. V. Galaktionov, M. S. Kalenkov, and A. D. Zaikin, Phys. Rev. B **77**, 094520 (2008).

[22] Z. Pajović, M. Božović, Z. Radović, J. Cayssol, and A. Buzdin, Phys. Rev. B **74**, 184509 (2006).

[23] K. Halterman, O. T. Valls, and P. H. Barsic, Phys. Rev. B **77**, 174511 (2008).

[24] M. Eschrig and T. Lofwander, Nat. Phys. **4**, 138 (2008).

[25] G. Eilenberger, Z. Phys. **214**, 195 (1968).

[26] A. I. Larkin and Yu. N. Ovchinnikov, in *Nonequilibrium Superconductivity*, edited by D. N. Langenberg and A. I. Larkin (Elsevier, Amsterdam, 1984).

[27] R. S. Keizer, S. T. B. Goennenwein, T. M. Klapwijk, G. Miao, G. Xiao, and A. Gupta, Nature (London) **439**, 825 (2006).

[28] I. Sosnin, H. Cho, V. T. Petrashov, and A. F. Volkov, Phys. Rev. Lett. **96**, 157002 (2006).

[29] M. D. Lawrence and N. Giordano, J. Phys.: Condens. Matter **8**, 563 (1996).

[30] M. Giroud, H. Courtois, K. Hasselbach, D. Mailly, and B. Pannetier, Phys. Rev. B **58**, R11872 (1998).

[31] V. T. Petrashov, I. A. Sosnin, I. Cox, A. Parsons, and C. Troadec, Phys. Rev. Lett. **83**, 3281 (1999).

[32] J. Aumentado and V. Chandrasekhar, Phys. Rev. B **64**, 054505 (2001).

[33] V. V. Ryazanov, V. A. Oboznov, A. Yu. Rusanov, A. V. Veretennikov, A. A. Golubov, and J. Aarts, Phys. Rev. Lett. **86**, 2427 (2001).

[34] T. Kontos, M. Aprili, J. Lesueur, F. Genêt, B. Stephanidis, and R. Boursier, Phys. Rev. Lett. **89**, 137007 (2002).

[35] Y. Blum, A. Tsukernik, M. Karpovski, and A. Palevski, Phys. Rev. Lett. **89**, 187004 (2002).

[36] A. Bauer, J. Bentner, M. Aprili, M. L. Della Rocca, M. Reinwald, W. Wegscheider, and C. Strunk, Phys. Rev. Lett. **92**, 217001 (2004).

[37] A. F. Volkov, F. S. Bergeret, and K. B. Efetov, Phys. Rev. Lett. **90**, 117006 (2003).

[38] M. Houzet and A. I. Buzdin, Phys. Rev. B **76**, 060504(R) (2007).

[39] V. Braude and Yu. V. Nazarov, Phys. Rev. Lett. **98**, 077003 (2007).

[40] J. Linder and A. Sudbø, Phys. Rev. B **76**, 064524 (2007).

[41] T. Champel, T. Löfwander, and M. Eschrig, Phys. Rev. Lett. **100**, 077003 (2008).

[42] A. F. Volkov, Ya. V. Fominov, and K. B. Efetov, Phys. Rev. B **72**, 184504 (2005); Ya. V. Fominov, A. F. Volkov, and K. B. Efetov, *ibid.* **75**, 104509 (2007).

[43] A. F. Volkov, A. Anishchanka, and K. B. Efetov, Phys. Rev. B **73**, 104412 (2006).

[44] P. SanGiorgio, S. Reymond, M. R. Beasley, J. H. Kwon, and K. Char, Phys. Rev. Lett. **100**, 237002 (2008).

[45] N. B. Kopnin, *Theory of Nonequilibrium Superconductivity* (Clarendon, Oxford, UK, 2001).

[46] A. V. Zaitsev, Sov. Phys. JETP **59**, 1015 (1984).

[47] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Phys. Rev. B **64**, 134506 (2001).

[48] A. A. Golubov, M. Y. Kupriyanov, and E. Il'ichev, Rev. Mod. Phys. **76**, 411 (2004).

[49] I. Baladie and A. Buzdin, Phys. Rev. B **64**, 224514 (2001).

[50] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Phys. Rev. B **65**, 134505 (2002).

[51] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Phys. Rev. B **75**, 184510 (2007).

[52] D. Y. Gusakova, A. A. Golubov, M. Y. Kupriyanov, and A. Buzdin, JETP Lett. **83**, 327 (2006).

[53] M. Yu. Kupriyanov and V. F. Lukichev, Sov. Phys. JETP **67**, 1163 (1988).

[54] E. A. Demler, G. B. Arnold, and M. R. Beasley, Phys. Rev. B **55**, 15174 (1997).

[55] T. Kontos, M. Aprili, J. Lesueur, and X. Grison, Phys. Rev. Lett. **86**, 304 (2001).

[56] L. Cretinon, A. K. Gupta, H. Sellier, F. Lefloch, M. Faure, A. Buzdin, and H. Courtois, Phys. Rev. B **72**, 024511 (2005).

[57] M. Zareyan, W. Belzig, and Yu. V. Nazarov, Phys. Rev. Lett. **86**, 308 (2001).

[58] K. Halterman and O. T. Valls, Physica C **420**, 111 (2005).

[59] T. Yokoyama and Y. Tanaka, Phys. Rev. B **75**, 132503 (2007).

[60] A. F. Volkov and K. B. Efetov (unpublished).